Signal Processing for GPS/IRNSS Receiver

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Overview

Plan

Objective of the Project

- Design and implement the most essential signal processing blocks required in building an IRNSS receiver
- Throughly analyze the blocks to identify key engineering parameters and study their inter-dependencies, as the first step towards subsequent development of an ASIC receiver for IRNSS

The Approach Taken

- Identify the similarities between GPS and IRNSS
- Use the vast GPS literature and available GPS front-end units, to build and study GPS signal processing blocks
- Identify the differences between GPS and IRNSS, and adapt the GPS implementation to IRNSS

The Three Blocks

- Signal Acquisition
- Signal Tracking
- Psuedo-range Computation

Plan

Trilateration

Trilateration: Locate any point in space given three reference points.

Let (x_i, y_i, z_i)
be the known coordinates
of the satellite i and (x_r, y_r, z_r) be the unknown receiver coordinates.
If the distances ρ_i can be found then the distance equations can be solved for (x_r, y_r, z_r)

For three satellite, $\quad i = 1, 2, 3$

$$\rho_i = \sqrt{(x_i - x_r)^2 + (y_i - y_r)^2 + (z_i - z_r)^2}$$

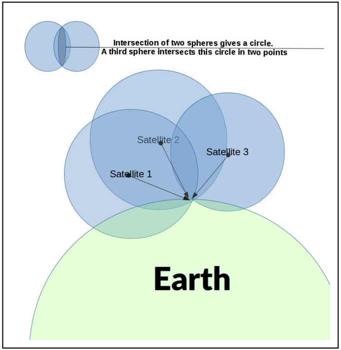


Figure: *Trilateration Illustration*

Time and Space References

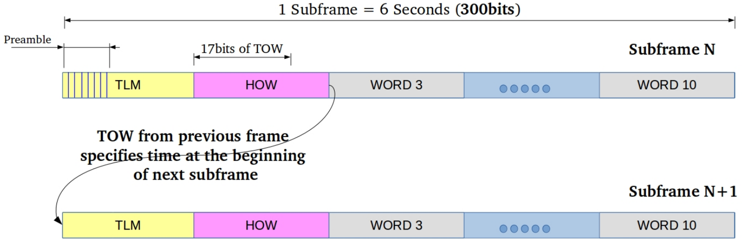
Setting Up the System

- Have satellites with accurately known orbits as references
- Have a common synchronized time reference
- Transmit signature signals simultaneously at commonly agreed times and also convey the satellite location at transmission
- Receiver identifies and separates the signals, and measures the delay t_i in arrival of signals from each of the satellites
- Multiply t_i with speed of EM Waves c and get **psuedo-ranges** $\rho_i = ct_i$. Solve the range equations to get user receiver position

Plan

The Navigation Data

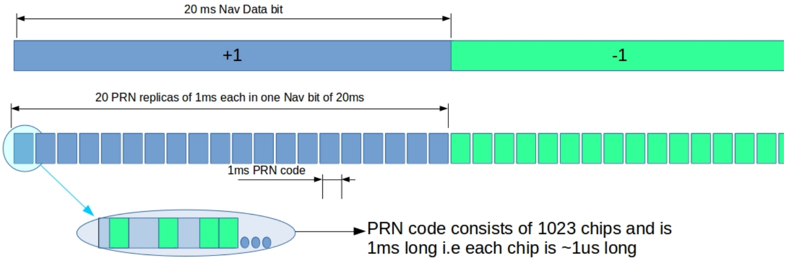
Navigation Bits $D(t)$ carry information about Ephemeris Data, satellite almanacs which includes time of transmission information, clock corrections and ionospheric models. They are transmitted at the rate of **50 bits per second**



Preamble = 10001011
TLM = Telemetry Word HOW = Hand-Over Word
TOW= Time Of Week (measured in units of 1.5 sec)
First 17 bits of HOW = most significant 17 bits of TOW

The Signature Signals

Psuedo-Random Noise (PRN) sequences $x(t)$ are bit streams with low auto correlation and cross-correlation properties. 1023 bits 'Gold Code' sequences lasting 1ms are used. Each satellite has a different code sequence.

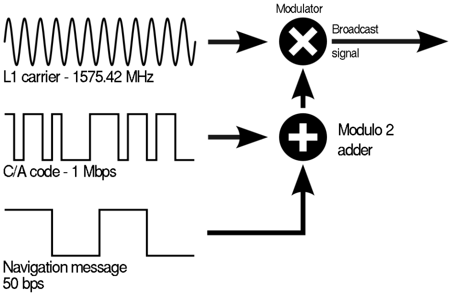


The PRN codes are multiplied (modulo 2 addition in terms of bits) to obtain the final signal to be modulated onto the carrier using Binary Phase Shift Keying (BPSK)

The Transmitted Signal

The bit sequences are BPSK modulated on to a carrier of frequency f_c, and transmitted as RHCP signals.

$$s_{f_{L1}}(t) = \sqrt{2P}D(t)x(t)cos(2\pi f_c t)$$
$$D(t) = \text{NAV Data bits}$$
$$x(t) = \text{PRN codes}$$



Content

The Receiver Clock

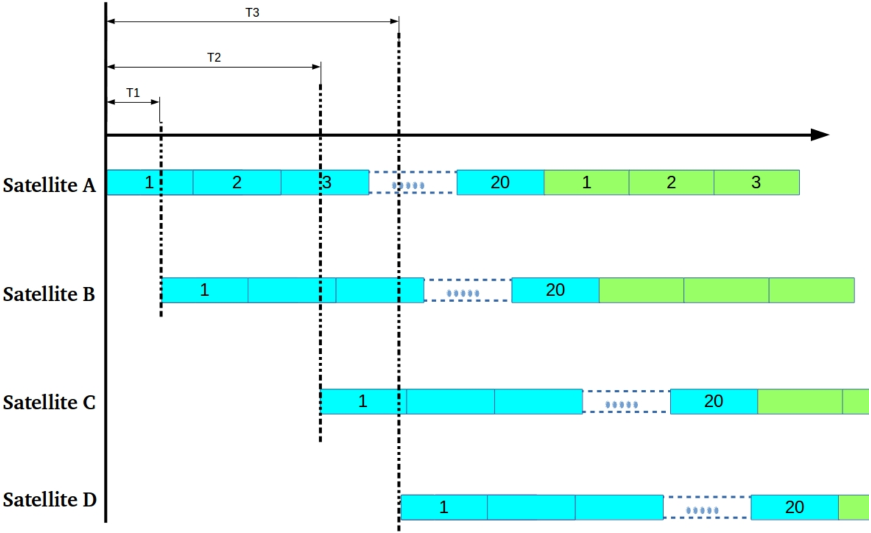
Receiver has a less accurate clock, not synchronized to GPS time

$$\rho_i = c(t_i + \Delta t) = \sqrt{(x_i - x_r)^2 + (y_i - y_r)^2 + (z_i - z_r)^2}$$

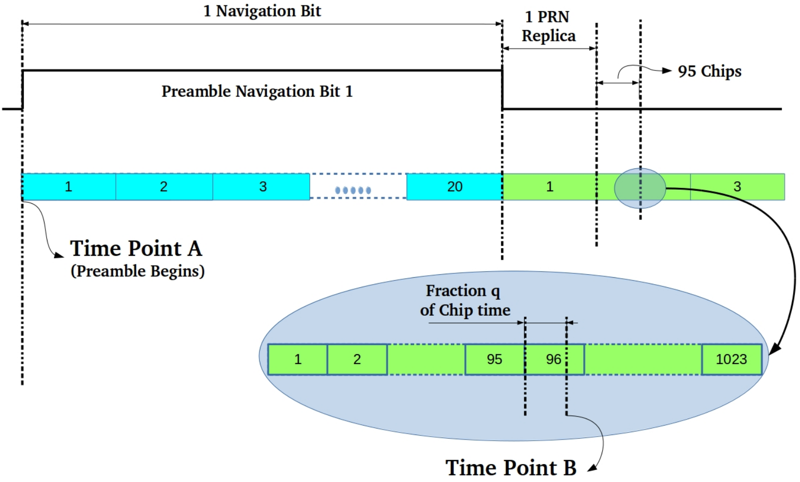
Thus we need 4 visible satellites. For $i = 2, 3, 4$

$$\begin{aligned}\rho_1 - \rho_i = &\sqrt{(x_1 - x_r)^2 + (y_1 - y_r)^2 + (z_1 - z_r)^2} \\ &- \sqrt{(x_i - x_r)^2 + (y_i - y_r)^2 + (z_i - z_r)^2}\end{aligned}$$

Relative Delay Measurement



Relative Delay Measurement



Time between point A and B = (1 Nav Bit) X (20ms) + (1 PRN) X (1ms) + (95 Chips) X (1/1023)ms + (q) X (1/1023)ms

Plan

The Received Signal

The signal has undergone Doppler shift and additive noise by the time it is received. Let f_d be the unknown Doppler shift and τ be the net time of travel of the signal. Then the received signal $s_r(t)$ is,

$$s_r(t) = \sqrt{2P_r}D(t-\tau)x(t-\tau)cos(2\pi\hat{f}t+\theta)+n(t)$$

$$\begin{aligned}
\text{where } \tau &= \text{the `code-phase' at reception },\\
\hat{f} &= f_{L1} + f_d,\\
\theta &= \text{unknown carrier phase at reception}\\
n(t) &= \text{additive noise}
\end{aligned}$$

Content

The Brute-force Search

Consider a time interval of length T_{int}, which is a multiple of 1ms, the length of PRN codes. $s(t)$ is co-related with a local signal generated according to receiver's estimate, say $\{\hat{f}_d, \hat{\tau_c}\}$. Co-relation is computed as:

$$\mathfrak{R}_I(\hat{f}_d, \hat{\tau_c}) = \int_0^{T_{int}} s(t)\, x(t - \hat{\tau_c})\, \cos(2\pi(f_{IF} + \hat{f}_d)t)\, dt$$

$$\mathfrak{R}_Q(\hat{f}_d, \hat{\tau_c}) = \int_0^{T_{int}} s(t)\, x(t - \hat{\tau_c})\, \sin(2\pi(f_{IF} + \hat{f}_d)t)\, dt$$

Let, $\mathfrak{R}(\hat{f}_d, \hat{\tau_c}) = \mathfrak{R}_I + j\mathfrak{R}_Q$, where $j = \sqrt{-1}$,

$$\|\mathfrak{R}(\hat{f}_d, \hat{\tau_c})\|^2 = \mathfrak{R}_I^2 + \mathfrak{R}_Q^2$$

Acquisition

$\|\mathfrak{R}\|^2$ is known as the ambiguity function. Aim of acquisition is to search over the space of $\hat{f}_d, \hat{\tau}_c$ to find a distinctly high value of $\|\mathfrak{R}\|^2$. Thus the output of acquisition is,

$$\{\tilde{f}_d, \tilde{\tau}_c\} = \underset{\{\hat{f}_d, \hat{\tau}_c\}}{\operatorname{argmax}} \quad \|\mathfrak{R}(\hat{f}_d, \hat{\tau}_c)\|^2$$

With $\delta_{f_d} = \hat{f}_d - f_d$

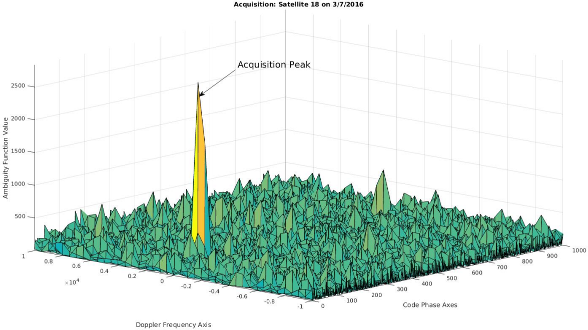
$$\therefore \mathfrak{R}(\hat{f}_d, \tau_c) \approx (De^{j\theta}\sqrt{P_r})\,\frac{\sin(\pi\delta_{f_d} T_{int})}{2\pi\delta_{f_d}}\, e^{\pi\delta_{f_d} T_{int}} + \widetilde{\mathfrak{n}}$$

The Doppler shift in frequency is known to be in within $\pm$6kHz. Hence the frequency search space is $-6000 \leq \hat{f}_d \leq 6000$. And since the PRN sequence lasts 1ms, code-phase search space is $0\text{ms} \leq \hat{\tau}_c < 1\text{ms}$

Content

Acquisition

Receiver discrete-izes the search space with a frequency bin size of say Δf_d and code-phase bin of $\Delta \tau_c$. The ambiguity function is only computed for every Δf_d^{th} frequency and every $\Delta \tau_c^{th}$ code-phase shift. And hence the estimates $\{\tilde{f}_d, \tilde{\tau_c}\}$ thus obtained are 'coarse'.



Acquisition

Acquisition Parameters

- T_{int}: the coherent integration time
- Δf_d: the frequency bin size
- $\Delta\tau_c$: the code-phase bin size

T_{int} and Δf_d

Receiver multiplies incoming $\cos(2\pi(f_{IF} + f_d)t + \theta)$ with $\cos(2\pi(f_{IF} + \hat{f}_d)t)$ and $\sin(2\pi(f_{IF} + \hat{f}_d)t)$.
Resulting signal will be $\cos(2\pi(f_d - \hat{f}_d)t + \theta)$ and $-\sin(2\pi(f_d - \hat{f}_d)t + \theta)$

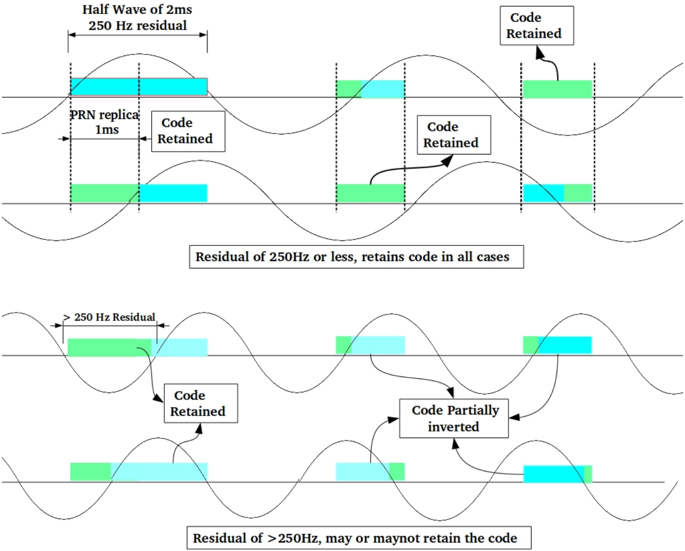
The Residual Carrier

Thus a residual carrier of $(f_d - \hat{f}_d)$ Hz will remain in the two signals, before we correlate with the PRN code to wipe-off the code
Choose Δf_d, such that,

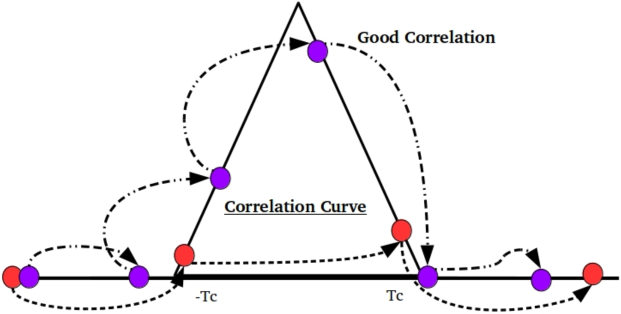
$$(f_d - \hat{f}_d) \leq \frac{\Delta f_d}{2} \leq \frac{1}{(4T_{int})}$$
$$\implies \Delta f_d \leq \frac{1}{(2T_{int})}$$

But the general thumbrule is $\Delta f_d \leq \frac{2}{(3T_{int})}$, based on acquisition loss in $\|\Re(\hat{f}_d, \hat{\tau}_c)\|^2$

T_{int} and Δf_d



$\Delta\tau_c$



$\Delta\tau_c$

Take $\Delta\tau_c \approx T_c$. With sampling rate of F_s, take $\Delta\tau_c = \frac{1}{F_s}\left\lceil \frac{T_c}{F_s} \right\rceil$

Plan

Need for Tracking

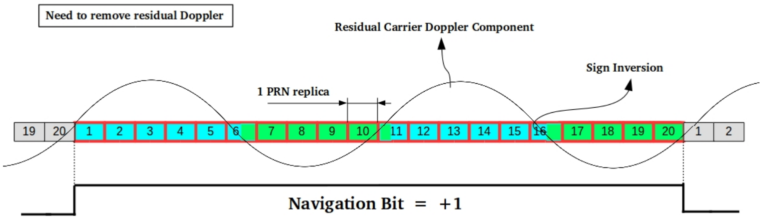
After Acquisition

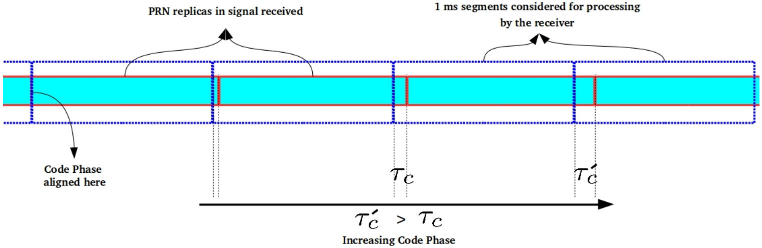
If acquisition detects the correct peak, the receiver concludes the estimates $\{\tilde{f}_d, \tilde{\tau_c}\}$, such that,

$$|\tilde{f}_d - f_d| \leq \frac{\Delta f_d}{2} \tag{1}$$

$$|\tilde{\tau_c} - \tau_c| \leq \frac{\Delta \tau_c}{2} \tag{2}$$

Need for Tracking





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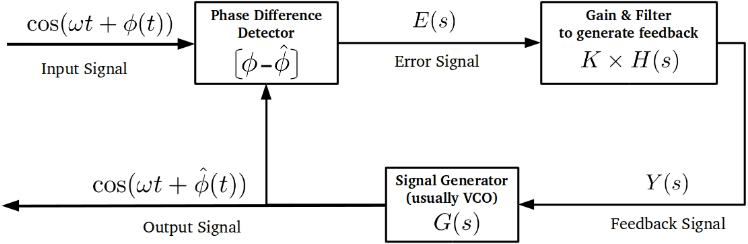


$$\lim_{\delta_{f_d},\theta\to 0} \mathfrak{R}(\hat{f}_d, \tau_c)$$

$$= \lim_{\delta_{f_d},\theta\to 0} D\frac{\sin(\pi\delta_{f_d} T_{int})}{\pi\delta_{f_d}} e^{j(\theta+\pi\delta_{f_d} T_{int})}$$

$$= \pm D$$

The Phase Lock Loop

$$2\cos(2\pi(f_{IF} + \hat{f}_d)t + \hat{\theta})\cos(2\pi(f_{IF} + f_d)t + \theta) \rightarrow \cos(2\pi(f_d - \hat{f}_d)t + \theta - \hat{\theta})$$

$$-2\sin(2\pi(f_{IF} + \hat{f}_d)t + \hat{\theta})\cos(2\pi(f_{IF} + f_d)t + \theta) \rightarrow \sin(2\pi(f_d - \hat{f}_d)t + \theta - \hat{\theta})$$

Let $\Delta\omega = 2\pi(f_d - \hat{f}_d)$ and $\Delta\theta = \theta - \hat{\theta}$.

$$\begin{aligned} I &= \int_{T_1}^{T_1+T} \cos(\Delta\omega t + \Delta\theta)\, dt \\ &= \frac{2}{\Delta\omega}\sin\left(\frac{\Delta\omega T}{2}\right)\cos\left(\Delta\omega\frac{(2T_1 + T)}{2} + \Delta\theta\right) \\ Q &= \int_{T_1}^{T_1+T} \sin(\Delta\omega t + \Delta\theta)\, dt \\ &= \frac{2}{\Delta\omega}\sin\left(\frac{\Delta\omega T}{2}\right)\sin\left(\Delta\omega\frac{(2T_1 + T)}{2} + \Delta\theta\right) \end{aligned}$$

Now to extract the phase-difference, we use the tan^{-1} discriminator. It is immune to 180° phase shifts.

$$e_r = tan^{-1}\left(\frac{Q}{I}\right) = \text{sign}\left(\frac{Q}{I}\right)\left(\left|\Delta\omega\frac{(2T_1+T)}{2}+\Delta\theta\right| \mod \frac{\pi}{2}\right)$$

If $T_1 = 0$, we get the error term $e_r = \frac{(\Delta\omega T)}{2} + \Delta\theta$.

PLL Update Rules

We use the update rules,

$$\hat{\theta} = \hat{\theta} + K_1 e_r$$
$$\hat{f}_d = \hat{f}_d + K_2 e_r$$

The PLL: Analysis

The PLL operation can be modeled as a discrete time system,

$$\theta[n] = \theta[n-1] + \omega[n-1]T$$
$$e_r[n] = \frac{(\omega[n] - \hat{\omega}[n])}{2}T + \theta[n] - \hat{\theta}[n]$$
$$\hat{\theta}[n] = \hat{\theta}[n-1] + \hat{\omega}[n-1]T + K_1 e_r[n-1]$$
$$\hat{\omega}[n] = \hat{\omega}[n-1] + K_2 e_r[n-1]$$

The PLL: Analysis

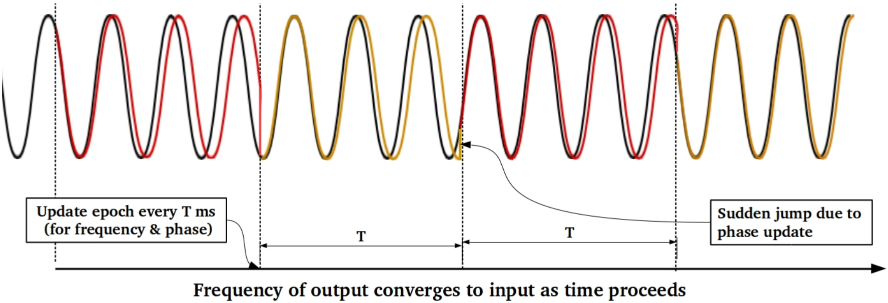
The PLL operation can be modeled as a discrete time system, Let $E_r(z), \Theta(z), \hat{\Theta}(z), \Omega(z), \hat{\Omega}(z)$ be the respective z transforms. Taking z transform and substituting, we get,

$$E_r(z) = \frac{(z^2 - 1)}{2z^2 + (2K_1 + L - 4)z + (L + 2 - 2K_1)} \Omega(z)\, T$$

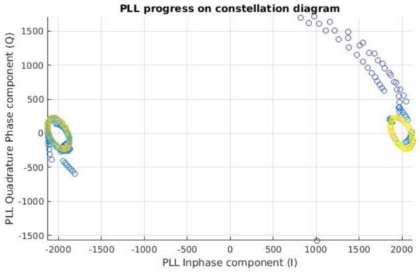
where $L = K_2 T$. Let $H(z) = 2z^2 + (2K_1 + L - 4)z + (L + 2 - 2K_1)$. The poles of this second order system are roots of $H(z)$, which are,

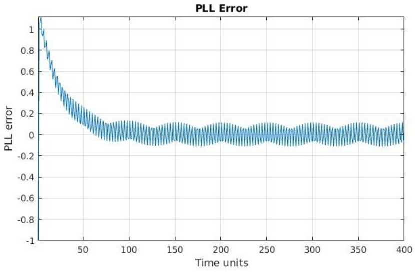
$$\frac{-(2K_1 + L - 4) \pm \sqrt{(2K_1 + L - 4)^2 - 8(L - 2K_1 + 2)}}{4}$$

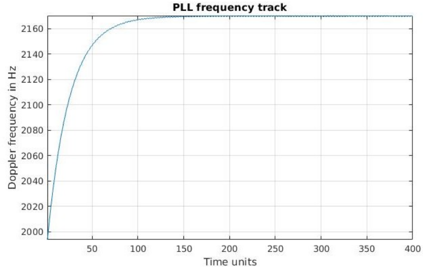
The PLL: Analysis

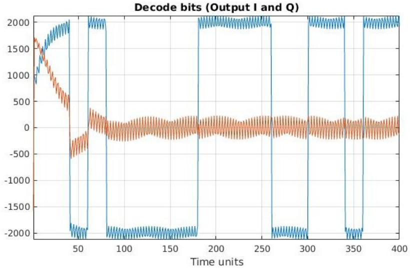


The PLL: Analysis





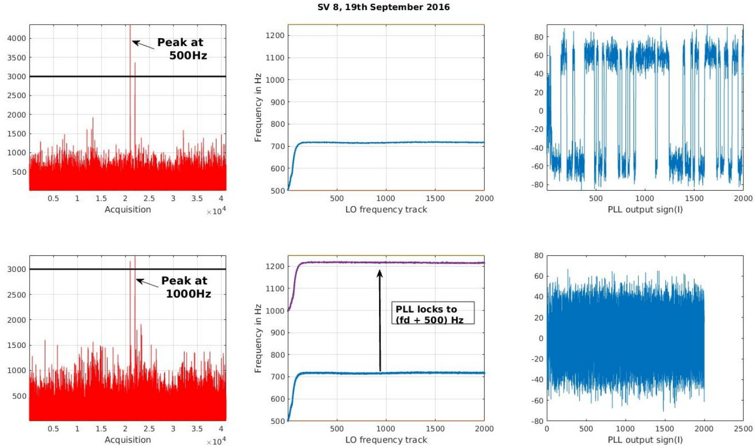




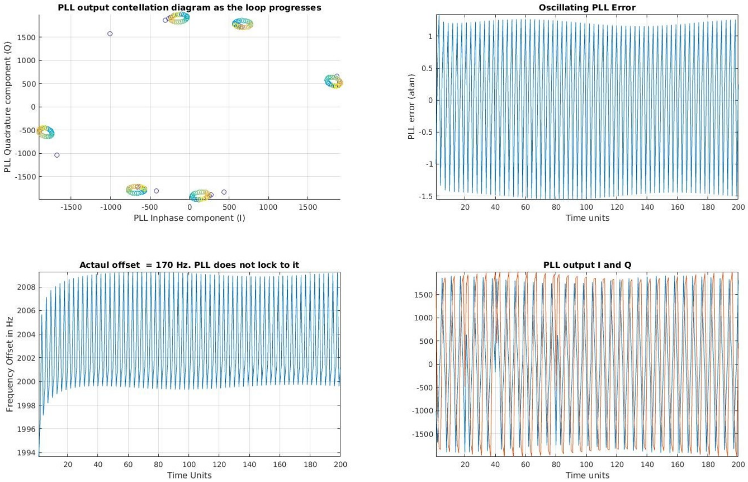
Content

False PLL Lock

The discriminator output is the same for $\Delta\omega + \frac{\pi}{T}k$ for any integer k. That means, the PLL can not distinguish between f_d and $f_d + \frac{1}{2T}$.



Since the $\tan^{-1}$ discriminator outputs $\left(\frac{(\Delta\omega T)}{2} + \Delta\theta\right) \mod \frac{\pm\pi}{2}$ rather than just $\left(\frac{(\Delta\omega T)}{2} + \Delta\theta\right)$, the error wraps around $\frac{\pm\pi}{2}$.



NAV Bit Flips

Consider the following example:

$\theta = 0.4\,\pi, \hat{\theta} = 0$ and $\omega = 510, \hat{\omega} = 500$. Let $T = 1\text{ms}$, $K_1 = 1$.

Feedback $= \frac{2\pi(50T)}{2} + 0.4\pi = 0.45\pi$

$\therefore \hat{\theta} = 0 + 2\pi(500)T + 0.45\pi = 1.45\pi$

While $\theta = 0.4\pi + 2\pi(510)T = 1.5\pi$.

$\theta - \hat{\theta} \approx 0$.

Clearly, in the next few iterations the two signals will phase lock.

A little noise can flip its sign and result in the feedback -0.45π instead of 0.45π. Then,

$\hat{\theta} = 0 + 2\pi(500)T - 0.45\pi = 0.55\pi$

$\theta - \hat{\theta} \approx \pi$.

NAV Bit Flips

	PREAMBLE		PARITY BITS	TOW		SF ID	
SF 1	1 0 0 0 1 0 1 1	0 0 0 0 1 1 1 0 0 1 1 0 1 1 0 0	1 1 1 0 0 0	1 0 1 0 1 1 1 0 1 1 0 0 0 0 1 0 0	0 1	0 0 1 1 1 0 0	1 1
SF 2	1 0 0 0 1 0 1 1	0 0 0 0 1 1 1 0 0 1 1 0 1 1 0 0	1 1 1 0 0 0	1 0 1 0 1 1 1 0 1 1 0 0 0 0 1 0 1	0 1	0 1 0 1 0 1 0	0 1
SF 3	1 0 0 0 1 0 1 1	0 0 0 0 1 1 1 0 0 1 1 0 1 1 0 0	1 1 1 0 0 0	1 0 1 0 1 1 1 0 1 1 0 0 0 0 1 1 0	0 1	0 1 1 1 1 0 1	0 1
SF 4	1 0 0 0 1 0 1 1	0 0 0 0 1 1 1 0 0 1 1 0 1 1 0 0	1 1 1 0 0 0	1 0 1 0 1 1 1 0 1 1 0 0 0 0 1 1 1	0 1	1 0 0 1 0 0 1	0 0
SF 5	0 1 1 1 0 1 0 0	1 1 1 1 0 0 0 1 1 0 0 1 0 0 1 1	0 0 0 1 1 1	0 1 0 1 0 0 0 1 0 0 1 1 1 0 1 1 1	1 0	0 1 0 1 0 0 1	1 0

Figure: SV 16, 6th May 2017, NAV bit frames, first 65 bits of each frame

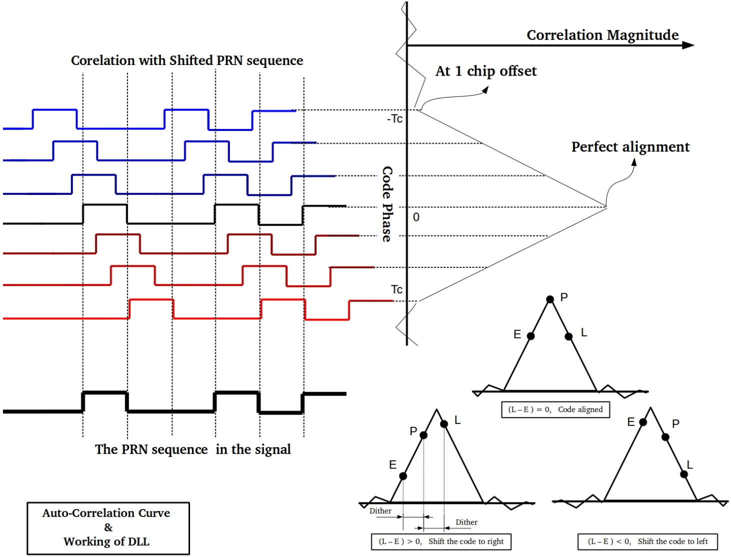
Possible solutions are:

- Use larger T to reduce the effect of noise. But increasing T involves additional computational costs.
- Cap the maximum absolute feedback value to say 0.25π. In this case, a switch in sign would have given -0.25π instead of 0.25π and caused a net error of 0.5π in the phase estimate.

Content

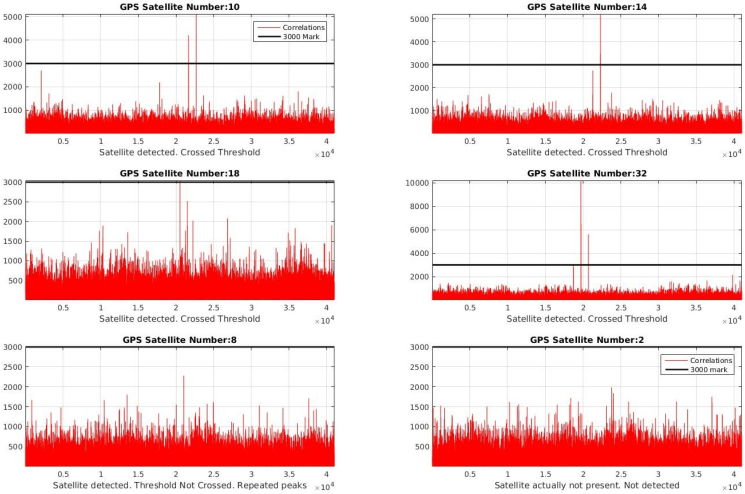
Delay Lock Loop

The DLL tracks the PRN code independent of the PLL.

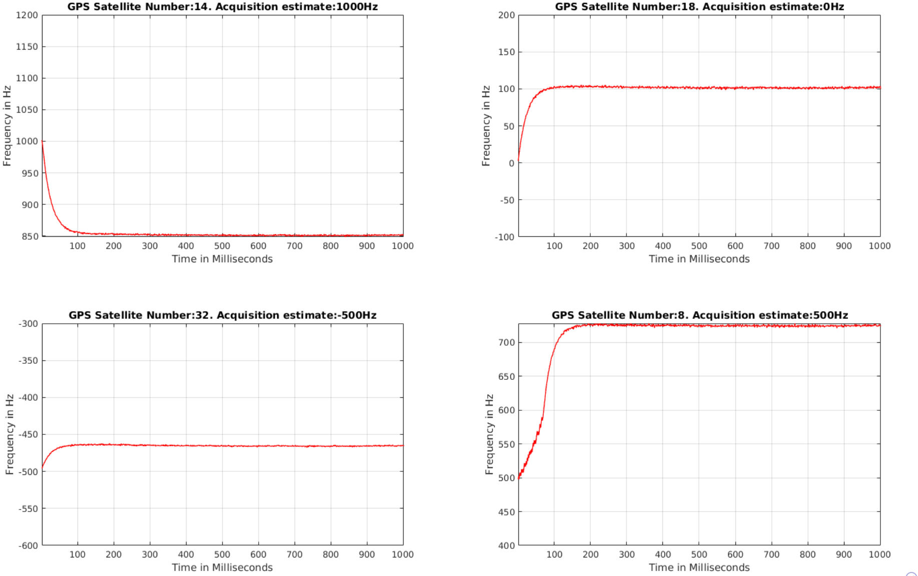


Plan

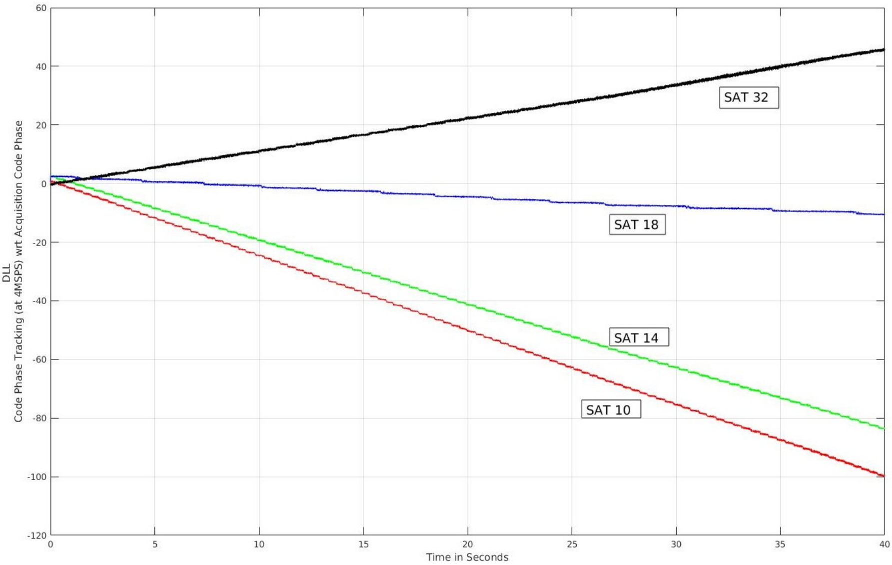
Acquisition



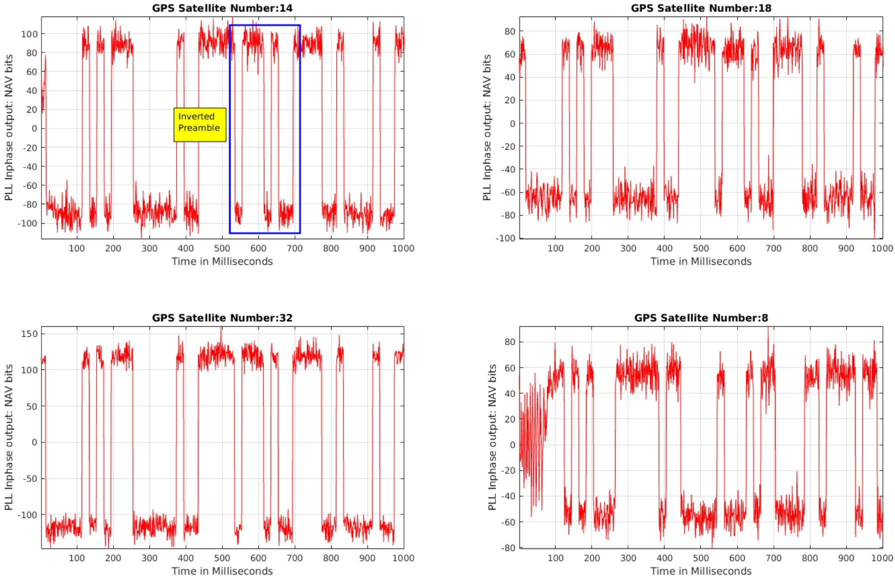
Frequency Tracking



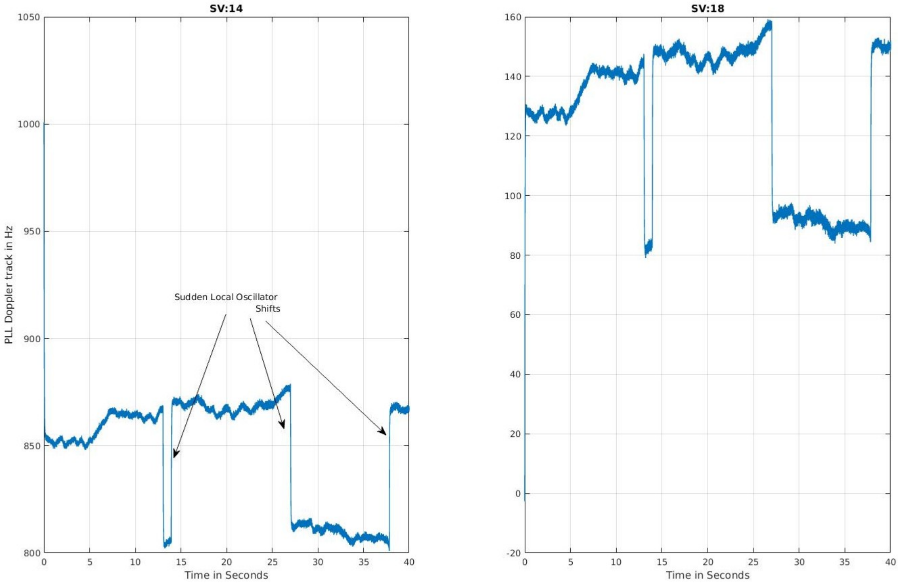
Code Phase Tracking



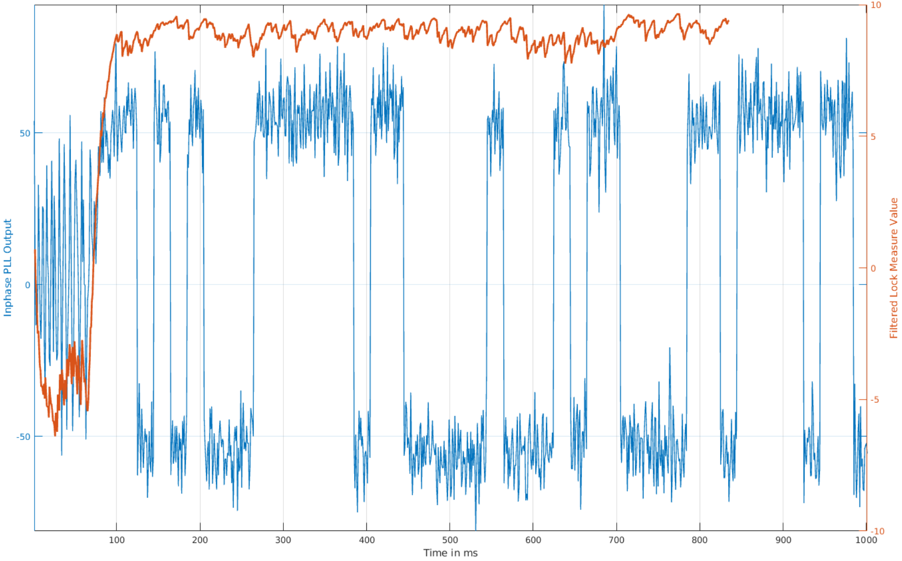
Navigation Bits



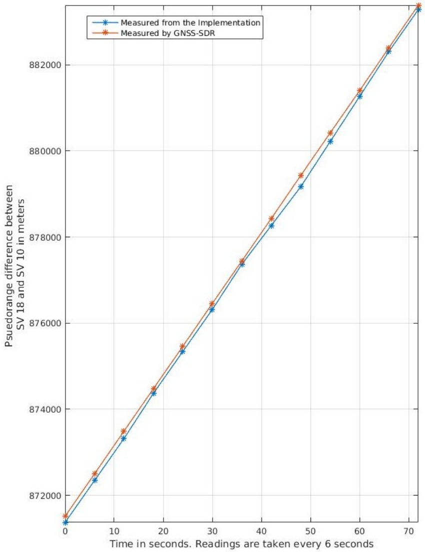
PLL response to sudden shifts

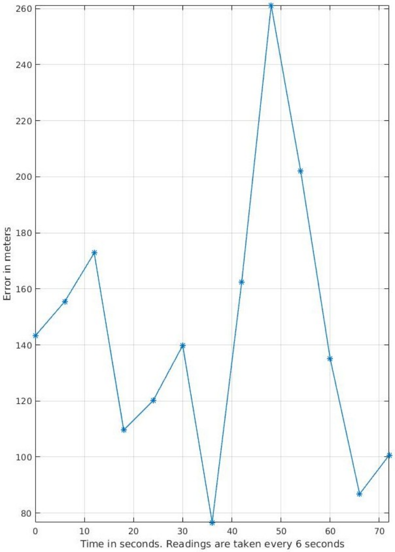


PLL Lock Measure



Psuedo-Range Accuracy





NAV Bit Frames

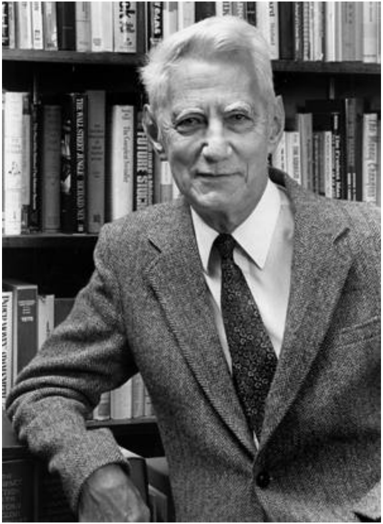
	PREAMBLE																								PARITY BITS					
SF 1	1	0	0	0	1	0	1	1	0	0	0	0	1	1	0	1	1	1	1	0	1	1	0	0	0	0	1	1	1	1
SF 2	1	0	0	0	1	0	1	1	0	0	0	0	1	1	0	1	1	1	1	0	1	1	0	0	0	0	1	1	1	1
SF 3	1	0	0	0	1	0	1	1	0	0	0	0	1	1	0	1	1	1	1	0	1	1	0	0	0	0	1	1	1	1
SF 4	1	0	0	0	1	0	1	1	0	0	0	0	1	1	0	1	1	1	1	0	1	1	0	0	0	0	1	1	1	1
SF 5	1	0	0	0	1	0	1	1	0	0	0	0	1	1	0	1	1	1	1	0	1	1	0	0	0	0	1	1	1	1
SF 1	1	0	0	0	1	0	1	1	0	0	0	0	1	1	0	1	1	1	1	0	1	1	0	0	0	0	1	1	1	1
SF 2	1	0	0	0	1	0	1	1	0	0	0	0	1	1	0	1	1	1	1	0	1	1	0	0	0	0	1	1	1	1
SF 3	1	0	0	0	1	0	1	1	0	0	0	0	1	1	0	1	1	1	1	0	1	1	0	0	0	0	1	1	1	1
	TLM WORD																													

	TOW (must be XORed with 1)																		SF ID (XOR with 1)										
SF 1	1	1	0	1	0	0	1	1	1	0	1	0	0	0	0	1	1	1	0	1	1	0	0	1	1	0	1	1	0	0
SF 2	1	1	0	1	0	0	1	1	1	0	1	0	0	0	0	1	0	1	0	1	0	1	0	0	0	0	0	1	0	0
SF 3	1	1	0	1	0	0	1	1	1	0	1	0	0	0	0	0	1	1	0	1	0	0	0	1	1	1	0	1	0	0
SF 4	1	1	0	1	0	0	1	1	1	0	1	0	0	0	0	0	0	1	0	0	1	1	0	0	1	1	0	0	0	0
SF 5	1	1	0	1	0	0	1	1	1	0	0	1	1	1	1	1	1	1	0	0	1	0	1	1	1	1	0	0	0	0
SF 1	1	1	0	1	0	0	1	1	1	0	0	1	1	1	1	1	0	1	0	1	1	0	0	0	0	0	1	1	0	0
SF 2	1	1	0	1	0	0	1	1	1	0	0	1	1	1	1	0	1	1	0	1	0	1	1	0	0	1	1	1	0	0
SF 3	1	1	0	1	0	0	1	1	1	0	0	1	1	1	1	0	0	1	0	1	0	0	0	0	0	1	0	1	0	0
	HOW WORD																													

	WEEK NUMBER									
SF 1	1	1	0	1	1	1	1	0	1	1
SF 2	0	1	0	0	1	0	0	0	0	0
SF 3	1	1	1	1	1	1	1	1	1	1
SF 4	0	1	1	1	1	0	1	0	1	0
SF 5	0	1	0	1	0	0	1	1	0	1
SF 1	1	1	0	1	1	1	1	0	1	1
SF 2	0	1	0	0	1	0	0	0	0	0
SF 3	1	1	1	1	1	1	1	1	1	1

Figure: NAV bit frames from SV 32, 19th Sepetember 2016, first 70 bits of each frame are shown. Verify that the Week Number in decimal is 891. The SF ID and TOW increment by 1 every subframe.

“I just wondered how things were put together”



Thank You!